Quantum-state information retrieval in a Rydberg-atom data register

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We analyze a quantum search protocol to retrieve phase information from a Rydberg-atom data register using a subpicosecond half-cycle electric field pulse. Calculations show that the half-cycle pulse can perform the phase retrieval only within a range of peak field values. By varying the phases of the constituent orbitals of the Rydberg wave packet register, we demonstrate coherent control of the phase retrieval process. By specially programming the phases of the orbitals comprising the initial wave packet, we show that it is possible to use the search method as a way to synthesize single energy eigenstates.

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I. INTRODUCTION

Two recent experiments have demonstrated data storage and retrieval using a quantum phase in Rydberg wave packets [1,2]. This was motivated by suggestions that multimode quantum interference can lead to protocols (''algorithms'') for efficient data storage and retrieval [3–10]. In one paper, a terahertz ''half-cycle'' electric field pulse retrieved a state that was marked only by its relative quantum phase [2]. Here, we discuss how the short half-cycle pulse (HCP) retrieves this phase information in a Rydberg atom. We present calculations and experiments that show that when the width of such a half-cycle pulse is much shorter than the Kepler orbital period of the wave packet (i.e., in the impulse approximation), the terahertz pulse tends to amplify the phase-flipped orbital of a Rydberg wave packet data register. This phase-to-amplitude conversion process can be explained by multimode interference. Further, the terahertz pulse has a certain range of peak field values for which it performs this retrieval task with highest fidelity.

We discuss the scaling of the phase-to-amplitude conversion with the size of the register and with the principal quantum number of the Rydberg states. We present the results of experiments to demonstrate the controlled input and retrieval of this phase information. We show that the quantum search for a single marked state in a quantum data register is an example of a coherent control process that transforms a superposition of quantum states to a single energy eigenstate.

II. SEARCHING WITH A HALF-CYCLE PULSE

In the simplest Rydberg atom implementation of the phase retrieval algorithm, the wave packet is the data register, and each of the constituent orbitals acts as an item in the database [2,11]. Information is encoded in the relative phases of the orbitals, as follows: all but one orbital have the same phase relative to a reference state (say phase 0), and the ''marked'' orbital has its phase opposite to the others (phase π). Thus an initial wave packet with the constituent orbitals $|24p\rangle$ to $|29p\rangle$ of cesium, of equal amplitude but with the phase of the $|26p\rangle$ orbital reversed with respect to the others, is represented by the column vector $(1,1,-1,1,1,1)^T/\sqrt{N}$, where $N=6$.

The action of the half-cycle pulse on the initial wave packet can be written (in atomic units) as

$$|\Psi(T)\rangle = \exp\left(-i\int_0^T dtE(t)z\right)|\Psi(0)\rangle. \tag{1}$$

In the impulse limit, when the pulse width is much smaller than the Kepler orbital period, the electronic state does not evolve while the pulse is on. The pulse transfers a momentum Q to the electron, equal to the time-integrated electric force:

$$|\Psi(T)\rangle = \exp(iQz)|\Psi(0)\rangle. \tag{2}$$

The matrix elements of the impulse operator in the energy basis can be written as

$$M_{n'\ell',n,\ell} = \langle n',\ell',m=0|\exp(iQz)|n,\ell,m=0\rangle. \tag{3}$$

For the database states ($24p$ to $29p$ of Cs),

$$M_{n'p,np} = \langle n',p|\exp(iQz)|n,p\rangle. \tag{4}$$

The exponential operator can be expanded in powers of Qz. Odd powers of Qz cannot connect states of the same parity, and do not contribute to these matrix elements. The terms with even powers of Qz can be written as a function F of Q^2z^2. The expression above can therefore be rewritten as

$$M_{n'p,np} = \delta_{n',n} - \langle n',p|F(Q^2z^2)|n,p\rangle. \tag{5}$$

Grouping the diagonal and off-diagonal terms together, we get

$$M_{n'p,np} = \delta_{n',n} - \langle n,p|F(Q^2z^2)|n,p\rangle - \langle n'\neq n,p|F(Q^2z^2)|n,p\rangle. \tag{6}$$

The behavior of this operator $M_{n'p,np}$ can be understood by examining its Q dependence, using the expression [2]

$$M_{np0}^{n'\ell'm'} = \langle R_{n'\ell'}(r)|i^{\ell'-1}\sqrt{\frac{3}{2\ell'+1}}[\ell' j_{\ell'-1}(Qr) - (\ell'-1)j_{\ell'+1}(Qr)]|R_{np}(r)\rangle\delta_{m',0}. \tag{7}$$

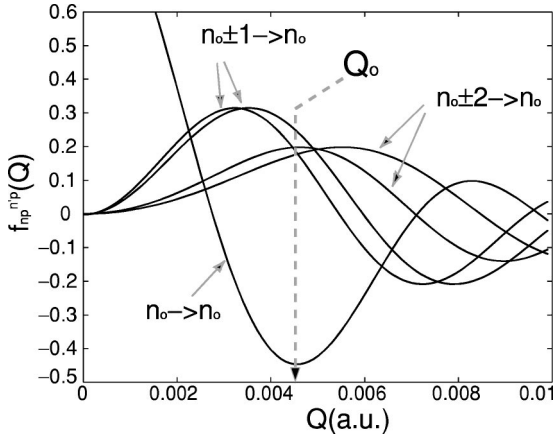

FIG. 1. Calculated amplitudes of the transition matrix elements that connect the $|n_0,p\rangle$ state to the $|n_0\pm 2,p\rangle$, $|n_0\pm 1,p\rangle$, and $|n_0,p\rangle$ states for $n_0=26$ as a function of Q the momentum transferred by an impulsive terahertz half-cycle pulse. The arrow indicates the value of Q_0 where Eq. (8) is satisfied.

The field-free single-electron wave functions are calculated using the Numerov algorithm with the appropriate quantum defect [2].

Calculated values of the transition matrix elements M are shown in Fig. 1 as a function of Q when $26p$ is the marked state. The diagonal matrix elements (which connect a database state to itself) and the off-diagonal matrix elements (which connect a database state to its neighbors in the database) both oscillate with Q. At $Q=0$, the diagonal matrix element is a maximum, while the off-diagonal matrix elements are zero. For a range of Q values around Q_0, the diagonal elements of the M matrix have the opposite sign from the off-diagonal elements. In Eq. (6), for $Q\simeq Q_0$, the diagonal term has the opposite sign to the off-diagonal terms. The n dependence of the matrix elements varies slowly for the states in our database, so we can model the general behavior of this system if we substitute $-a$ for the diagonal terms and b for the off-diagonal terms, to construct a simple matrix operator for the impulse on database states. The initial state transforms as follows:

$$\psi_f^{db}=\begin{bmatrix} -a & b & b & b & b & b \\ b & -a & b & b & b & b \\ b & b & -a & b & b & b \\ b & b & b & -a & b & b \\ b & b & b & b & -a & b \\ b & b & b & b & b & -a \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}\frac{1}{\sqrt{N}}. \tag{8}$$

If one of the states is marked, i.e., has its phase reversed with respect to those of the others, the multimode interference conditions are appropriate for constructive interference to the marked state, and destructive interference to the others. That is, the population in the marked state is amplified. This operator produces a contrast in the probability density of the final states of the database of $[a+(N-1)b]^2$ for the marked state versus $[a-(N-3)b]^2$ for the unmarked states, where N is the number of states in the data register. This procedure for data contrast enhancement is related to an ''inversion-about-mean'' operation (substitute $-a=1-2/N$ and $b=-2/N$) [3,12].

Numerical calculations using methods described in Ref. [2] show that the contrast produced by an HCP acting on an $N=6$ wave packet register with equal-amplitude orbitals is 0.35:0.004. The database population after the search is smaller than the initial population because the HCP transfers small-but-finite populations to the nondatabase states. This, however, does not significantly affect the outcome of the search process, because at the peak HCP field where the p-state database search is most effective, transfer to other angular momentum states (the nondatabase states) is very small. In the experimental field ionization spectra, the signal at energies corresponding to the nondatabase states is negligibly small. In the experiment, the orbitals do not have the same amplitudes, but have a Gaussian profile centered about the central wave packet energy. However, the action of the HCP on this wave packet still produces a clear phase-to-amplitude conversion, and this can be seen in both the experiment and calculations.

III. DEPENDENCE ON THE PEAK HCP FIELD

Figure 1 shows that constructive multimode interference occurs if the phase of an $|n_0,p\rangle$ state is programmed to be 180° out of phase from the rest of the states. The amplification of the marked state is greatest when the magnitude of the impulse is Q_0, but the conditions for constructive interference are met for a range of Q values. Therefore, the phase retrieval from a Rydberg wave packet is expected to be relatively insensitive to the precise value of Q, the total momentum transferred by the HCP. Q can be adjusted by changing the peak HCP field while the width remains constant. The experimental result for the $|26p\rangle$ flipped case is shown in Fig. 2(a). There is a band of values of Q for which the retrieval occurs successfully for each marked state. We calculate the phase retrieval spectra for the same marked state as a function of Q by solving the Schrödinger equation in an essential states basis [2]. The basis states are calculated using a grid-based pseudopotential method [13]. The results of the impulse and full calculations are shown in Figs. 2(b) and 2(c). Figure 2 shows a good qualitative agreement between the experiment and calculations. The discrepancies could arise due to contributions from transverse focusing effects [14], and the fact that the HCP field was not perfectly homogeneous due to the Gouy phase shift [15].

IV. SCALING OF THE PHASE RETRIEVAL

The performance of the phase retrieval procedure with increasing database size is predicted by the calculated results shown in Fig. 3. As the number of items in the database increases, the marked-state amplification produced by the HCP improves, tending to a value of 4. This behavior is in agreement with the simple model of the phase retrieval, presented in Eq. (1) of Ref. [2]. To excite more Rydberg states with an optical pulse, the wave packet should be centered at

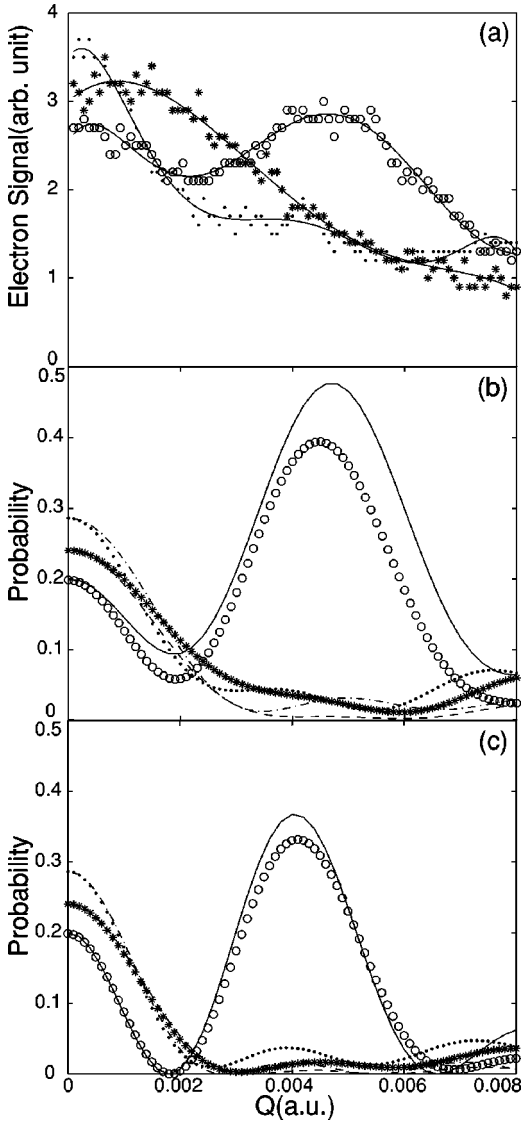

FIG. 2. Electron populations of the flipped state and the other database states after the interaction with the half-cycle pulse as a function of the peak value of the HCP field. (a) Experiment. At a certain value of the peak HCP field, the population in the phase flipped 26*p* state (circles) is clearly amplified compared to the 27*p* (dots) and 25*p* (stars) states. (b) Calculated values of the above. As the *p* and *d* states of Cs are not resolved experimentally, the populations of the combined *p* and *d* states are displayed. Calculations are done using both an impulse model (solid line, flipped 26*p*; dotted line, 27*p*; and dashed line, 25*p* states) and by numerically solving the Schrödinger equation (open circles, flipped 26*p*; filled circles, 27*p*; and stars, 25*p* states). (c) Same calculations as in (b), only the populations in the *p* ''database'' states are shown.

a higher principal quantum number, where the energy level spacing is smaller. Therefore, we investigate the scaling of the peak HCP with a change in *n*, the principal quantum number of the marked state. The phase retrieval is carried out by the impulse operator $\exp(iQz)$. In momentum space, this operator shifts the wave functions so that they interfere correctly to produce the phase-to-amplitude conversion. The momentum-space wave functions for a hydrogenlike atom

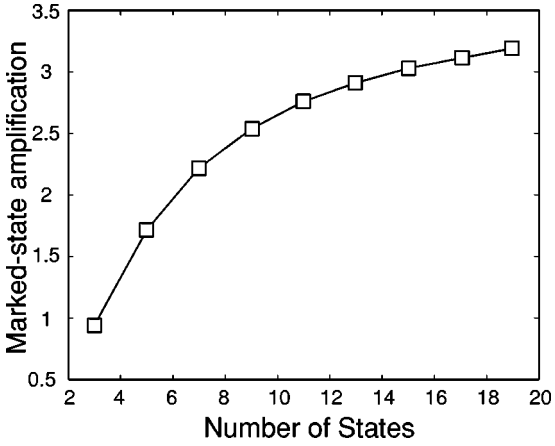

FIG. 3. Ratio between the populations in the marked state of the wave packet after and before the HCP as a function of increasing register size. Numerical calculations show that this marked-state amplification is enhanced with larger database size, and approaches a value of 4.

scale as n^2 [17]. Therefore, for marked states with different *n* values, the *Q* of the HCP that retrieves them can be expected to scale as n^{-2}. This is seen in Fig. 4, which shows the calculated values of peak HCP strength Q_0 for various *n* values, scaled by the inverse-square of the principal quantum number of the marked state. In practice, there is a trade-off between the increasing size of the database, and the decreasing resolution in the field-ionization spectrum as *n* increases. This limits the size of the Rydberg register. The Rydberg system has limited scaling capability, but otherwise contains all of the features needed to execute the search algorithm [18].

In practice, the phase retrieval is not performed with the efficiency expected from the matrix *M* in Eq. (8). A major reason for this is that the far off-diagonal matrix elements are, in general, smaller than the near off-diagonal elements.

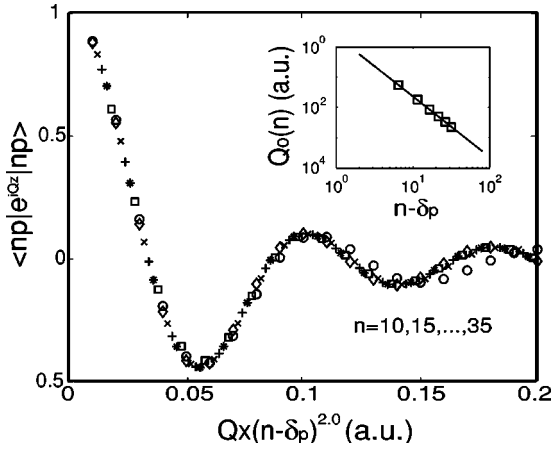

FIG. 4. Numerical calculations using a quantum defect show that the impulse Q_0 that creates the flipped-state amplification scales as the inverse square of the principal quantum number *n* of the marked state. Different symbols represent the calculations for different *n*'s: $n=10$ (○), 15 (×), 20 (+), 25 (*), 30 (◇), 35 (□).

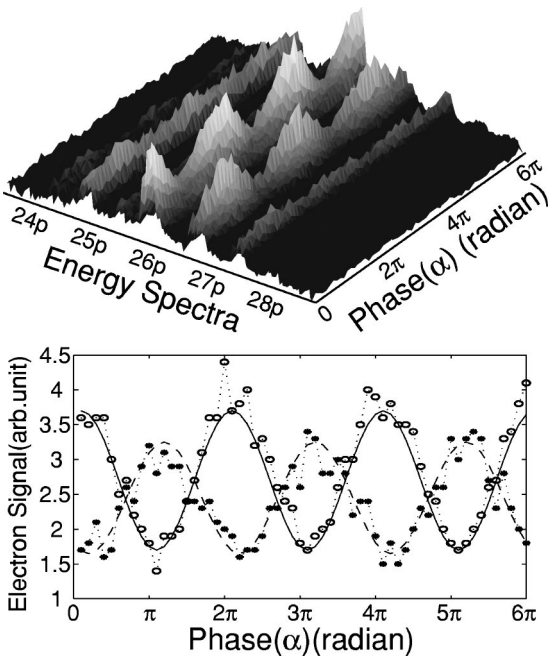

FIG. 5. A coherent control demonstration of the flipped-state amplification. The wave packet is initially prepared so that the HCP amplifies the $|n=26,p\rangle$ state population. Then the phases of the $|n=26,p\rangle$ and $|n=27,p\rangle$ states are rotated by α with respect to the other states. As this phase α changes from 0 through 6π, the flipped state alternates between $|n=26,p\rangle$ and $|n=27,p\rangle$ three times and the Rydberg population is seen to alternate accordingly.

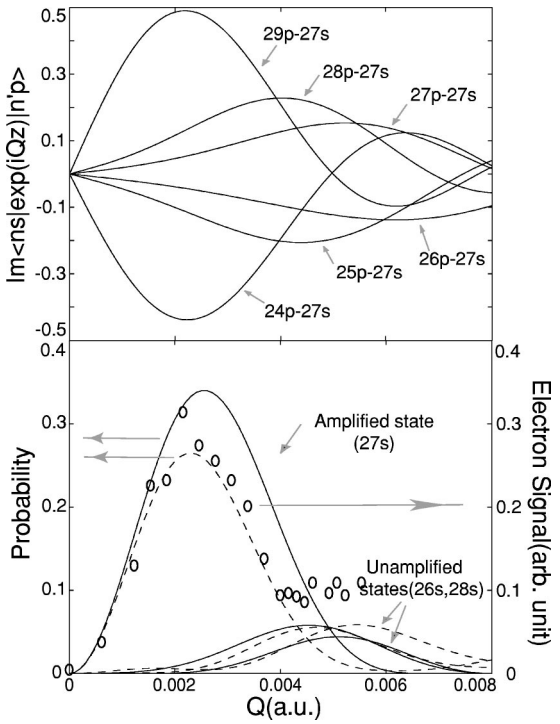

FIG. 6. (a) Matrix elements of the impulsive interaction of the HCP between the database states ($24p$ through $29p$) and the desired $27s$ state. By choosing the phases of the database states, a multimode interference condition is set up to produce an s state. (b) Population in the desired s state with increasing peak HCP field. The initial state is a superposition of $24p$ to $29p$ states of cesium. Open circles, experiment; solid line, impulse model calculation; dashed line, full calculation. Also shown are the populations of the $26s$ and $28s$ states which are not amplified by the HCP of $Q=0.0024$ a.u.

The amplification of population in the marked state occurs by transfer of population from the nearby states with a relative phase between them [19]. Since the overlap between states that are far apart in the register is small, the far off-diagonal terms in the matrix are also small. If the marked state is one of the outer states in the register, its phase retrieval is not as efficient as expected from the matrix. This effect was seen in the published experiment [2] where the $26p$ and $27p$ states were retrieved with higher fidelity than the $25p$ and $28p$ states. This problem has been addressed in a recent theoretical work [16], where, by using a shaped terahertz pulse designed by optimal control theory, it is shown that the phase retrieval can be performed successfully for all but the two outermost database orbitals.

V. EXPERIMENTAL DEMONSTRATION OF COHERENT CONTROL OF PHASE RETRIEVAL

A coherent control experiment was performed to demonstrate that the HCP probes the relative phase structure of the initial wave packet. Details about the wave packet preparation and generation of HCPs are given elsewhere [2]. We adjusted the peak terahertz field to optimally amplify a phase-reversed $26p$ state. The phase structure of the initial states was modified by shaping the ultrafast optical pulse that excites the wave packet. The spectral components of the optical pulse that excite the $|26p\rangle$ and $|27p\rangle$ states then were phase-shifted with respect to the other spectral components in such a way that the $26p$ and $27p$ states were always 180° out of phase with respect to each other. That is, the wave packet to be probed was

$$|\Psi\rangle=|24p\rangle+|25p\rangle-e^{i\alpha}|26p\rangle+e^{i\alpha}|27p\rangle+|28p\rangle+|29p\rangle. \tag{9}$$

At $\alpha=0$, the initial conditions are set so that $|26p\rangle$ is phase-reversed. If the phase shift (α) is 180°, we expect that the $|27p\rangle$ state should be amplified because the $|27p\rangle$ state is then the flipped state. The experimental result is shown in Fig. 5. The traces for the $|26p\rangle$ and $|27p\rangle$ state populations are exactly 180° out of phase with respect to each other, as designed in the experiment. This simple coherent control experiment confirms that the HCP probes the phase difference between the database states and that the information stored in the Rydberg data register is indeed in the flipped phase of the marked state. In a more complicated situation than the one considered in this paper, one could expand this phase study to any pair of excited energy states. Relative phases of all pairs of excited energy states provide the complete phase structure of the prepared wave packet.

VI. CREATING STATES OUTSIDE THE DATABASE

The terahertz field in the search algorithm performs a unitary transformation that concentrates the wave packet prob-

ability in a single basis state [20]. We explore this more general view by using the HCP to drive the amplitude into eigenstates other than those within the database. These outside states are initially unpopulated. For example, we can use the HCP to produce an *s* eigenstate, even though the initial ''database'' states are all *p* states. In the cesium spectrum, the *s* states are located between the *p* states of adjacent *n* manifolds because the difference in their quantum defects is approximately a half-integer. The phase conditions for the multimode interference to produce *s* states are different from the ''marked state'' phases that produce amplification of *p* states (register states). Figure 6(a) shows the transition matrix elements between the np ''database'' states and a particular $27s$ state (spectrally in between the $26p$ and $27p$ states). Clearly, to produce an *s* state, all the higher *p* states have to be in phase, but out of phase with all the lower *p* states. An impulse interacting with a wave packet that has this phase structure creates the constructive interference condition for that *s* state. The phase structure of the initial wave packet that produces the $|27s\rangle$ state (that is, in between the $|26p\rangle$ and $|27p\rangle$ states) is $(1,1,1,-1,-1,-1)$. The experimental spectra of the decoded wave packet as a function of the peak field of the HCP show that *s* states appear at an HCP strength of $Q=0.0024$ a.u., as seen in Fig. 6(b).

We note that the peak HCP field that produces an *s* state is different from that which produces a *p* state. To produce a desired eigenstate, it is nescessary to have not only the correct phase structure in the initial state, but also the appropriate peak HCP field. Calculations indicate that to produce an eigenstate of higher angular momentum, a higher HCP field is needed. This is consistent with the classical notion of an HCP providing a momentum ''kick'' to a bound electron which then acquires angular momentum proportional to the strength of the impulse.

From the viewpoint of wave packet creation, this result is noteworthy. Previous experiments have shown that a half-cycle pulse can interact with a Rydberg eigenstate to produce a wave packet [21] because of its broadband nature. This experiment shows that the inverse is also achievable—a properly programmed wave packet can be driven towards an eigenstate by a broadband HCP.

VII. CONCLUSIONS

We have shown the connections between the phase retrieval protocol in a Rydberg data register via an impulsive half-cycle pulse, and the more general problem of constructing a database-search algorithm [3]. The sensitivity of the phase retrieval to the peak HCP field is explained by multimode interference. We show how this implementation scales with increasing database size, increasing peak HCP field, and with increasing principal quantum number of the marked state. We demonstrate coherent control over the retrieval of phase information using an HCP. By programming the initial wave packet and the HCP, it is possible to create the correct interference conditions to produce any desired energy eigenstate.

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[1] J. Ahn, T.C. Weinacht, and P.H. Bucksbaum, Science **287**, 463 (2000).
[2] J. Ahn, D.N. Hutchinson, C. Rangan, and P.H. Bucksbaum, Phys. Rev. Lett. **86**, 1179 (2001).
[3] L.K. Grover, Phys. Rev. Lett. **79**, 325 (1997).
[4] E. Biham *et al.*, Phys. Rev. A **60**, 2742 (1999).
[5] J.C. Howell and J.A. Yeazell, Phys. Rev. Lett. **85**, 198 (2000).
[6] P.G. Kwiat *et al.*, J. Mod. Opt. **47**, 257 (2000).
[7] F. Mang, Phys. Rev. A **63**, 052308 (2001).
[8] L. C. Kwek, *et al.*, Phys. Lett. A **267**, 24 (2000).
[9] I.L. Chuang, N. Gershenfeld, and M. Kubinec, Phys. Rev. Lett. **80**, 3408 (1998).
[10] M.O. Scully and M.S. Zubairy, Proc. Natl. Acad. Sci. U.S.A. **98**, 9490 (2001).
[11] L.K. Grover, Phys. Rev. Lett. **79**, 4709 (1997).
[12] L.K. Grover, Phys. Rev. Lett. **80**, 4329 (1998).
[13] J. N. Bardsley, in *Case Studies in Atomic Physics*, edited by E. W. McDaniel and M. R. C. McDowell (North-Holland, Amsterdam, 1974), Vol. 4, p. 302; C. Rangan, K.J. Schafer, and A.R.P. Rau, Phys. Rev. A **61**, 053410 (2000).
[14] E. Budiarto, N.W. Pu, S. Jeong, and J. Bokor, Opt. Lett. **23**, 213 (1998).
[15] D. You and P.H. Bucksbaum, J. Opt. Soc. Am. B **14**, 1651 (1997).
[16] C. Rangan and P.H. Bucksbaum, Phys. Rev. A **64**, 033417 (2001).
[17] B. Podolsky and L. Pauling, Phys. Rev. **34**, 109 (1929).
[18] D.A. Meyer, Phys. Rev. Lett. **85**, 2014 (2000).
[19] L.K. Grover, Science **280**, 228 (1998).
[20] L.K. Grover, Phys. Rev. Lett. **85**, 1334 (2000).
[21] N.E. Tielking and R.R. Jones, Phys. Rev. A **52**, 1371 (1995).